begin to restrict the design. At this bandwidth, however, other filter configurations, such as the shunt-stub filter, may be a better choice.

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A Statistical Measure for the Stability of Solid State Noise Sources

MOTOHISA KANDA

Abstract—**A statistical measure for the stability of solid state noise sources is discussed. Its applicability is demonstrated. Two independent techniques, a cross correlation technique and a triangulation technique, are employed to estimate the instabilities of individual noise sources. An analysis for the validity of the cross correlation technique for separating the instability of the noise source from that of the measurement system is also given.**

I. INTRODUCTION

SOLID STATE noise sources are influenced by their environment, and an unsuitable environment can cause unstable operation. Observed instabilities can be traced to changes in ambient temperature, operating power supply voltages and currents, humidity, etc. In order to unravel the causes of these instabilities, the need for the measurement of the stability of solid state noise sources has received considerable attention [1].

The purpose of this paper is to describe a measure of stability applicable to solid state noise sources. Conventionally, the stability of devices such as amplifiers, signal generators, noise sources, etc., has been given in terms of the degree of changes or fluctuations in the observables for a specific sampling time interval. However, if one wishes to forecast future stabilities of a time series from present and past stability measurements, such a conventional measure for stability is obviously not sufficient, and a new statistical measure for stability is needed. For this purpose, Allan variance analysis, discussed in Section II, is used as a meaningful statistical measure for the stability of solid state noise sources.

Unless an experiment is carefully planned, the instabilities of the measurement system will often obscure those of the noise source under test, and a method for separating the instabilities is required. A technique similar to cross correlation is employed for this purpose and is described in Section 3.1. If a measurement process is used where the instability of the measurement system does not affect the measurement of the stability of the noise source under test, the triangulation technique, described in Section 3.2, will provide an estimate for the instability of noise from an individual noise source. These two independent techniques provide for a check on the validity of the individual measurement schemes used.

A comparison between the stability of the output noise power from commercial solid state noise sources and that from typical commercial argon gas noise sources is given in Section IV. The stabilities at several frequencies between 1 GHz and 18 GHz were examined. Two different manufacturers' sources and two models from each manufacturer that cover the frequency range are used. The noise diodes used for these commercial solid state noise sources are Read-type silicon avalanche diodes. The microwave noise generated by them is expected to be white and Gaussian. The long-term instability measurements over months and years have not been completed; however, the results shown in Section IV will give the statistical expectation for fluctuations of noise

Manuscript received September 15, 1976; revised January 27, 1977.
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and forecasting of the future fluctuations of noise from solid state and gas discharge noise sources. The results presented are also more directly usable where the output noise stability of less than one day is important (for example, the stability of reference noise for a noise-adding radiometer). A brief study made to identify the causes of instability in the solid state noise sources is also described in Section IV.

Mathematical models for analyzing the behavior of fluctuations of noise from noise sources and the measurement system are described in Section V. These models are used to check the validity of the cross correlation technique for separating the instability of the noise source from that of the measurement system.

II. Allan Variance Analysis

An important performance parameter for a solid state noise source is the stability of its mean output noise power, as a function of sampling time interval. For some models of the noise generation mechanisms, which seem to be representative of solid state noise sources, the classical variance is unbounded. A well-behaved and convergent stability measure is therefore needed. Such a measure has been developed for the field of frequency stability, and is called the Allan variance [2]–[4]. A special case of the Allan variance analysis [2] is used in this paper to establish a measure of stability for solid state noise sources.

A record of the phenomenon under consideration, $y(t)$, is divided into a number of equal time segments of length τ, and the average value of $y(t)$ of each segment, y_k, is calculated by

$$y_k = \frac{1}{\tau}\int_{t_k}^{t_k+\tau} y(t)\, dt \qquad (1)$$

where y_k is the kth-segment average, starting at time t_k. Next, a sample variance (sample size two), $\sigma_y^2(2,\tau)$, of successive averages is calculated. That is

$$\sigma_y^2(2,\tau) = \sum_{n=k}^{k+1} (y_n - \bar{y}_k)^2 = \tfrac{1}{2}(y_{k+1} - y_k)^2 \qquad (2)$$

where

$$\bar{y}_k \equiv \frac{1}{2}\sum_{n=k}^{k+1} y_n \qquad (3)$$

is the average of the two successive segment averages y_k and y_{k+1}. The Allan variance, $\sigma_y^2(\tau)$, for this special case (sample size two) is then defined to be [2]

$$\sigma_y^2(\tau) \equiv \langle \sigma_y^2(2,\tau)\rangle \qquad (4)$$

where the brackets represent the infinite time average of $\sigma_y^2(2,\tau)$ over all pairs of successive y_k constructed from $y(t)$. In practice, a finite set of $\sigma_y^2(2,\tau)$ often gives a good estimate of $\sigma_y^2(\tau)$.

In (1), y_k is the average of $y(t)$ in the full interval t_k to $t_k + \tau$. When t_{k+1} is coincident with, less than, or greater than $t_k + \tau$, we say the Allan variance of (2) has no dead time, negative dead time, or positive dead time, respectively. When y_k is an average of some, but not all, isolated values of $y(t)$ in the interval t_k to $t_k + \tau$, we say that the y_k values have distributed dead time. The Allan variance has not been defined for y_k values that have distributed dead time.

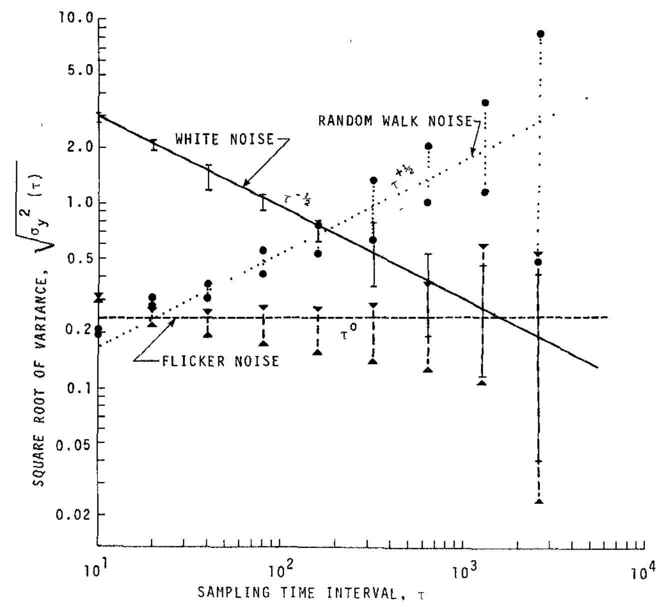

Fig. 1. Allan variances for three kinds of noise as a function of sampling time interval, τ.

The preceding calculation is repeated for various values of sampling time interval, τ. For a given maximum allowable rms deviation in $y(t)$, the maximum sampling time interval can then be determined. When the noise generation mechanism is random and unautocorrelated, the Allan variance, $\sigma_y^2(\tau)$, decreases according to the inverse of sampling time interval, τ. The data analysis is typically performed by a computer via a program designed to compute the appropriate Allan variance, $\sigma_y^2(\tau)$. In the computer program, logarithm $\sigma_y(\tau)$, versus logarithm τ is plotted.

One of the advantages of using the Allan variance analysis for a statistical measure of stability is that the Allan variance has a simple dependence on sampling time interval, τ, for common characteristics of fluctuations of noise. For example, when the power law spectral density, $S_y(f)$, of fluctuations of noise is white (i.e., $S_y(f) \propto f^0$), flicker (i.e., $S_y(f) \propto f^{-1}$), or random walk (i.e., $S_y(f) \propto f^{-2}$) in nature, the corresponding Allan variance, $\sigma^2(\tau)$, has sampling time interval, τ, dependences of τ^{-1}, τ^0, and τ, respectively [4], as illustrated in Fig. 1. It should also be noted that the analysis is not restricted to Gaussian distribution.

One disadvantage arises when the data are not taken continuously. The estimation for the Allan variance when the available data can give only y_k values with distributed dead time is quite complicated and has not been worked out completely. It is known [5], however, that ignoring the distributed dead time in the y_k values sometimes causes misleading results in Allan variance estimation. A discussion on the subject is given in Section V. Since estimating the Allan variance is quite difficult when distributed dead time is involved, it is advisable to choose the experimental methods which avoid distributed dead time in y_k values.

III. Measurement Techniques for Estimating the Stability of an Individual Noise Source

In general, the stability of noise intensity from a noise source can be measured only by comparing its output noise level with one or two other noise sources. Therefore, the direct result of any such measurement includes not only the instability of the noise source under test, but also that of a reference noise source. In addition, if a measurement system itself is not stable, the instability of the measurement system also affects the results of such a measurement. Two independent measurement schemes are used in this paper to estimate the stability of noise intensity from individual noise sources.

3.1. Cross Correlation Technique

When one uses a measurement scheme where its instability affects the measurement results, a cross correlation technique provides a means of separating the instability due to the noise source from that due to the measurement system.

The actual measurement system used is a total-power radiometer with a noise bandwidth of 2 MHz. Two independent noise sources, a solid state noise source and an argon gas discharge noise source, are used. The total-power radiometer is switched between the solid state noise source and the argon gas discharge noise source once every 6 s. Because of this slow switching, this total-power radiometer is relatively more sensitive to gain variations and, therefore, is more unstable than a switching radiometer, which will be discussed in Section 3.2.

The cross correlation technique is used to separate the instability due to the noise source from that of the total-power radiometer. If X_{sm} and Y_{gm} are, respectively, the time series of observables for the intensity of the output noises from a solid state noise source and from an argon gas discharge noise source, the cross correlation with zero-delay time, $R_{X,Y}(\tau)$, is defined to be

$$R_{X,Y}(\tau) \equiv \frac{\langle X_{sm} \cdot Y_{gm} \rangle}{\sqrt{\langle X_{sm}^2 \rangle \langle Y_{gm}^2 \rangle}} = \frac{\langle (x_{k+1} - x_k) \cdot (y_{k+1} - y_k) \rangle}{\sqrt{\langle (x_{k+1} - x_k)^2 \rangle \langle (y_{k+1} - y_k)^2 \rangle}} \tag{5}$$

where the brackets represent the "infinite" time average. Then, one can show that the Allan variance for the measurement system $\sigma_m^2(\tau)$ is given by

$$\sigma_m^2(\tau) = R_{X,Y}(\tau) \cdot \sigma_X(\tau) \cdot \sigma_Y(\tau) \tag{6}$$

and

$$\sigma_X^2(\tau) = \sigma_s^2(\tau) + \sigma_m^2(\tau) \tag{7}$$

$$\sigma_Y^2(\tau) = \sigma_g^2(\tau) + \sigma_m^2(\tau) \tag{8}$$

where $\sigma_s^2(\tau)$, $\sigma_g^2(\tau)$, and $\sigma_m^2(\tau)$ are, respectively, Allan variances for the solid state noise source, the argon gas discharge noise source, and the measurement system, which are assumed to be independent of and not cross correlated with each other. Thus by use of (6), it is possible to extract the instability due to a solid state noise source from that due to the measurement system.

In (5), X and Y have been measured as a pair of time series. However, a switched total-power radiometer X and Y are not strictly simultaneously obtained. Rather, measurements of X are alternated in time with measurements of Y. Then, using (2), variances σ_X^2 and σ_Y^2 using y_k values are calculated, which, in general, have varying amounts of distributed dead time. For the purpose of this paper, it is formally assumed that the values of this variance are equal to the values of the corresponding Allan variance with no dead time. A further discussion of this subject is given in Section V.

An approximation of σ_m^2 can be estimated by computating $R_{X,Y}(\tau)$ from our measured data and by using (6). Because the instability of the measurement system does not have a white spectrum, but instead, is divergent toward lower frequencies, (6) and this procedure are approximately valid. The analysis for the validity of (6) is discussed in Section V.

It should be noted that this technique, using cross correlation, is very powerful for estimating the stability of two-port active devices such as amplifiers, receivers, etc., as well as for estimating the stability of one-port devices such as noise sources.

3.2. Triangulation Technique

If one can use a measurement setup such as a switching radiometer where the measurement system is inherently stable, the technique described below is more attractive than the cross correlation technique described earlier. This type of radiometer samples power from one noise source and then from the other by switching repetitively from one to the other. The switching speed of this switching radiometer is typically 30 cps, which is about 180 times faster than that of our total-power radiometer. In addition, a null balance is usually achieved by adjustment to make the average noise output power from the two noise sources identical. With these two modifications, the measurement system is inherently stable and does not significantly affect the measurement of stability of a noise source.

Using the switching radiometer, the stability of fluctuation of noise intensity from a noise source can be measured by comparing its output with that of other noise sources. The resultant instability includes the instability of the noise source under consideration as well as that of the reference noise source. The separation of instabilities due to these two noise sources is quite simple. We use three independent noise sources and measure the Allan variance of one noise source with respect to the other. Then, for three independent noise sources, we have

$$\sigma_{a,b}^2(\tau) = \tfrac{1}{2}\sigma_a^2(\tau) + \tfrac{1}{2}\sigma_b^2(\tau) + \frac{1}{2M} \sum_{k=1}^{M} (\Delta y_{a,k} \Delta y_{b,k}) \tag{9}$$

where Δy_k equals $y_{k+1} - y_k$, the difference between adjacent noise measurement, and M is the number of differences. Similarly,

$$\sigma_{b,c}^2(\tau) = \tfrac{1}{2}\sigma_b^2(\tau) + \tfrac{1}{2}\sigma_c^2(\tau) + \frac{1}{2M} \sum_{k=1}^{M} (\Delta y_{b,k} \Delta y_{c,k}) \tag{10}$$

$$\sigma_{c,a}^2(\tau) = \tfrac{1}{2}\sigma_c^2(\tau) + \tfrac{1}{2}\sigma_a^2(\tau) + \frac{1}{2M} \sum_{k=1}^{M} (\Delta y_{c,k} \Delta y_{a,k}). \tag{11}$$

The cross-product term goes to zero as the number of measurements increases, provided the noise sources are independent of and not cross correlated with each other. Equations (9), (10), and (11) can be solved for an individual Allan variance. For example,

$$\sigma_a^2(\tau) = (\sigma_{a,b}^2(\tau) + \sigma_{c,a}^2(\tau) - \sigma_{b,c}^2(\tau)) - \frac{1}{2M} \sum_{k=1}^{M} (\Delta y_{a,k} \Delta y_{b,k} + \Delta y_{c,k} \Delta y_{a,k} - \Delta y_{b,k} \Delta y_{c,k}). \quad (12)$$

This triangulation technique is used as well as the cross correlation technique to estimate the stability of fluctuations of noise intensity from individual noise sources, and the experimental results are given in the next section.

IV. Experimental Results

Fig. 2 shows the results of the Allan variance analysis for a typical commercial solid state noise source. The square root of the Allan variance of $\delta T/T$, where T is the output radiation noise temperature from the solid state noise source, is 0.0014 for a 1-day sampling time interval. This corresponds to 0.006 dB, where the decibel is calculated from $10 \log_{10} (1 + \sqrt{\sigma_{\delta T/T}^2(\tau)})$. Since the Allan variance increases with an increase of sampling time interval, the noise generation mechanism for the solid state noise source is autocorrelated (correlated with itself). To compare the stability of the solid state noise source with that of a typical commercial argon gas discharge noise source, the square root of the Allan variance of a typical argon gas discharge noise source is also shown in Fig. 2. The square root of the Allan variance of $\delta T/T$ is about 0.0002 (or 0.001 dB) for a 1-day sampling time interval. The stability of the argon gas discharge noise source is found to be very good. In the experimental results shown in Fig. 2, we have used two independent techniques as indicated in Section III in order to estimate the stability of an individual noise source. The experimental results obtained by use of these two independent techniques agree with each other well within the uncertainties involved.

Although it is beyond the scope of this paper to describe the causes of instability in the solid state noise sources in detail, the effects due to two main environmental influences, viz., ambient temperature and supply current, are examined below.

The temperature dependence of the solid state noise source is shown in Fig. 3. It is obvious from the data that there is no measurable ambient temperature dependence of the output noise as long as a constant-current power supply is used to operate the solid state noise source.

The output noise from the solid state noise source as a function of the dc operating current is plotted in Fig. 4 using logarithmic scales. The solid line shows I^{-1} behavior, where I is the bias current, as is predicted in the theory of Hines [6]. These results suggest that the output noise approaches the predicted I^{-1} behavior except at extremely low bias currents, where a) the avalanche discharge in the solid state noise source becomes unstable and nonuniform, and therefore, b) the measurements lose their accuracy. Fig. 5 presents experimental results showing the dependence of the output noise on the operating current for several argon gas discharge noise sources. It is empirically found from Fig. 5 that the output noise from the argon gas discharge noise source follows an $I^{-0.1}$ law where I is its dc discharge current.

The stable operation of a solid state noise source is more strongly dependent on its operating current than is an argon

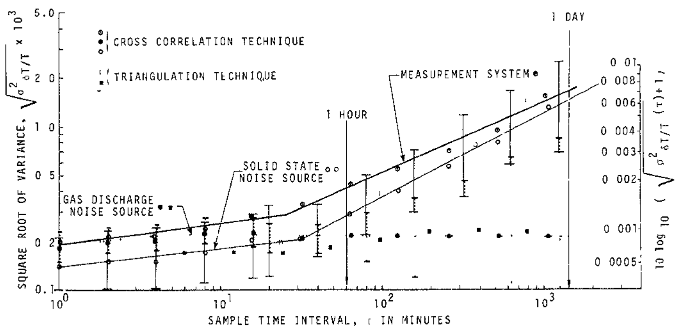

Fig. 2. Allan variances for fluctuations of noise intensity of noise sources as a function of sampling time interval, τ.

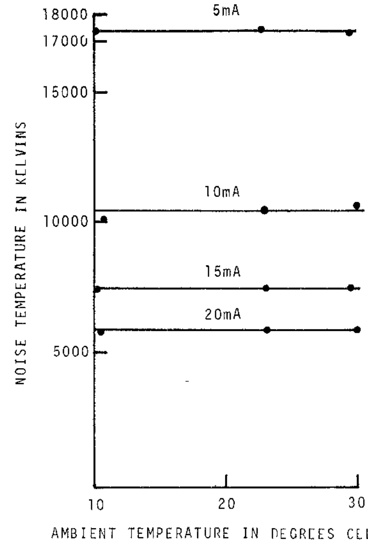

Fig. 3. Noise intensity of a solid state noise source as a function of ambient temperature.

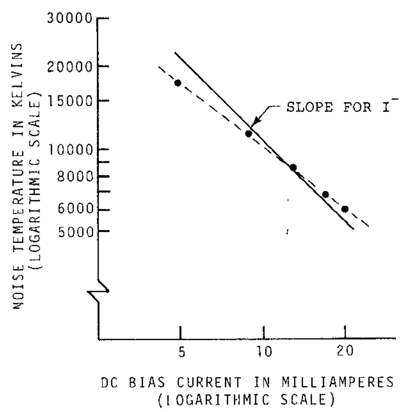

Fig. 4. Noise intensity of a solid state noise source as a function of dc bias current.

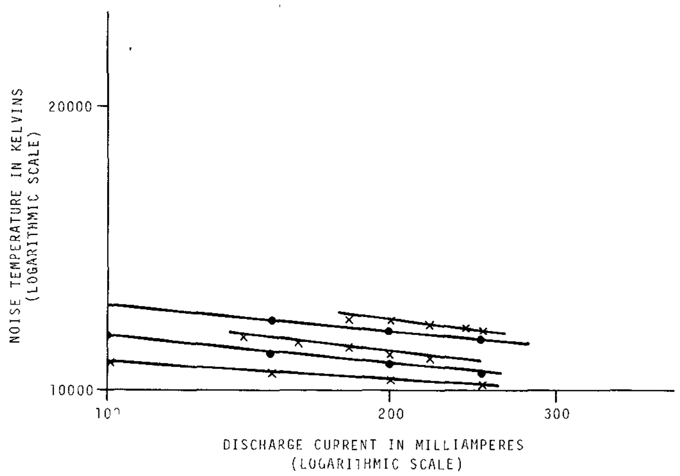

Fig. 5. Noise intensities of five different argon gas discharge noise sources as a function of dc discharge current.

gas discharge noise source. A stable current source should be used in the operation of solid state noise sources.

V. Discussions on Confidence in the Measurement Technique

In this section, we mainly discuss the validity of the cross correlation technique for separating the instability of the noise sources from that of the total-power radiometer and comparison of simulation with experiments.

5.1. Computer Simulations for Fluctuation of Noise Intensity of Noise Sources

Comparing Figs. 1 and 2 indicates that the fluctuations of noise from the solid state noise source and the measurement system can be approximated by the random walk process, whereas the fluctuations of noise from the argon gas discharge noise source can be approximated by the flicker noise process. Therefore, in order to study the fluctuations of noise intensity from the two noise sources via simulation, we have generated a discrete flicker noise process and two discrete random walk noise processes from independent white noise processes.

The mathematical model which generates a discrete sequence of numbers of a flicker noise is given by Barnes and Jarvis [7], and is used in this paper. The intensity of each noise process is scaled appropriately. For each simulation, 40 processes are generated with the confidence levels (3σ) indicated by vertical bars. Fig. 6 shows the square root of the Allan variances, $\sqrt{\sigma^2_{\delta T/T}(\tau)}$, for numerical modelings that simulate the fluctuations of noise from a solid state noise source and from an argon gas discharge noise source.

5.2. Influence of Distributed Dead Time on Allan Variances

The time series of observables, X_{sm} and Y_{gm}, which are, respectively, those for the fluctuations of noise intensities from the solid state noise source and from the argon gas discharge noise source, contaminated with noise of the gain of the measurement system, are measured alternately in time with each other. Therefore, the time series of observables,

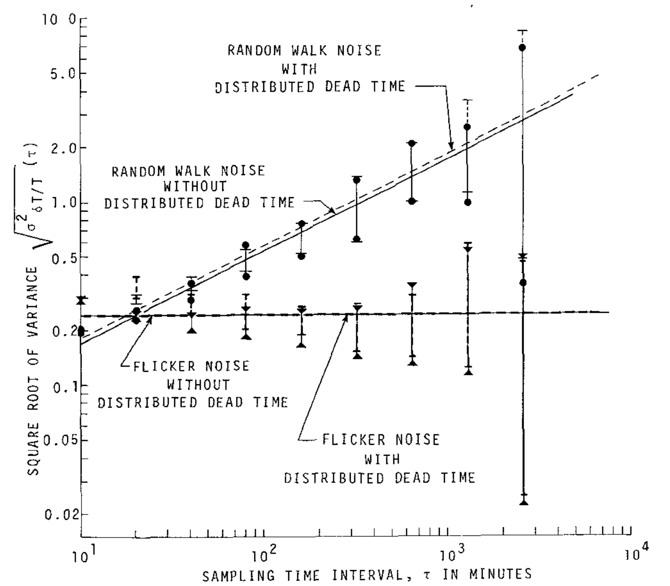

Fig. 6. Computer simulation of Allan variances using data averages with and without distributed dead time.

X_{sm} and Y_{gm}, contain distributed dead time in themselves. The theoretical development of general Allan variance analysis, which can take care of distributed dead time in the time series values, is complicated, and has not been published. However, it is found from simulations shown in Fig. 6, that for a flicker and a random walk noise process, no significant differences are found between Allan variances with distributed dead time and without distributed dead time. These results are intuitively understandable, particularly for a random walk process, since the time series value at the dead time for a random walk process can be best approximated by the adjacent value of the time series. Thus we conclude that, for the purpose of the statistical measure of stability using the Allan variance analysis, the time series of observables, X_{sm} and Y_{gm}, can be treated as though they do not contain any distributed dead time in themselves. However, no similar conclusion can be reached for the cross correlation function of (5) and (6), as discussed in the next section.

5.3. The Use of Cross Correlation Technique for Separating Instability of a Noise Source from that of the Measurement System

Finally, we are in a position to examine the validity of the cross correlation technique for separating the instabilities of the noise sources from that of the measurement system. In our experiments, instead of measuring the time series X_{sm} and Y_{gm} strictly simultaneously, the measurements of X_{sm} are alternated in time with the measurements of Y_{gm}. Thus the cross correlation between these two time series has a nonzero delay time in itself. It should be noted that, as the sampling time interval increases, the fractional delay time of the cross correlation becomes effectively smaller and smaller.

Suppose the gain variation of the total-power radiometer did have a white spectrum instead of a random walk spectrum. Then, the cross correlation between nonoverlap-

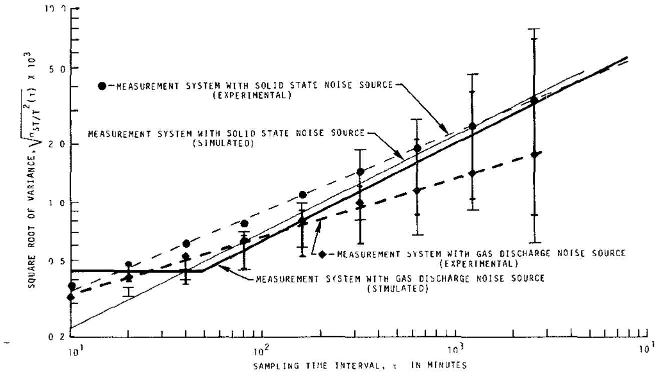

Fig. 7. Simulated Allan variances for fluctuations of noise intensities of a solid state noise source and of a gas discharge noise source, contaminated with noise of the gain of the measurement system.

ping values of X_{sm} and Y_{gm} would always be zero, and this technique would fail to separate the instability of a noise source from that of the total power radiometer. Fortunately, the gain variation of this measurement system does have a random walk spectrum, and this technique is valid.

To examine the validity of this technique for low frequency divergent noises, three independent noise processes are generated numerically from independent white noise processes. The first is a random walk noise process which represents the fluctuations of noise intensity from a solid state noise source. The second is a flicker noise process which represents the fluctuations of noise intensity from an argon gas discharge noise source. The third is a random walk noise process which represents the fluctuations of gain of the total-power radiometer.

By adjusting the intensity of each of these noise processes, the numerical time series X_{sm} and Y_{gm} are constructed. The results are shown in Fig. 7 along with experimental results. In order to simulate our experiments properly, these numerical time series contain distributed dead time, which was realized by eliminating every other point in the time sequence.

To estimate the uncertainty caused by ignoring distributed dead time, the following procedure was used. The cross correlation with minimum delay time is calculated using (5) and is shown in Fig. 8. Then, the cross correlation of the numerical time series, X_{sm} and Y_{gm}, which do *not* contain any distributed dead time in themselves, are calculated and shown in Fig. 8 along with the experimental results.

Fig. 8 indicates that the cross correlation factor with distributed dead time is estimated to be about 50 percent smaller than that without distributed dead time. That is, the estimate of the cross correlation by ignoring the distributed dead time is biased, and the bias of the estimate causes an error of as much as 100 percent.

To approximate the uncertainty in estimating individual Allan variances using the cross correlation technique, the following procedure was used. By use of (6) and the cross correlation with distributed dead time, we then estimate the Allan variance for the total-power radiometer, $\sigma_m^2(\tau)$, and corresponding Allan variance for the solid state noise source $\sigma_s^2(\tau)$ and that for the argon gas discharge noise source $\sigma_g^2(\tau)$ by use of (7) and (8). These resultant indirect estimates of Allan variances are compared in Fig. 9 with direct estimates of Allan variances for the numerical time series, X_{sm} and Y_{gm}.

The discrepancy between these two estimates of Allan variances shown in Fig. 9 contributes to bias and uncertainties in estimating individual Allan variances by use of the cross correlation technique. In principle, an evaluated bias can be used as a correction factor to arrive at the final estimate. However, because of an uncertainty due to the model, both bias and uncertainty are included as a total estimation uncertainty in this paper. It is found from Fig. 9 that the dispersion of 40 independent Allan variances increases with the increase of the sampling time interval, τ. For example, when one uses the data length of 82 h, estimation uncertainty of an Allan variance at 41-h sampling time, for the simulated random walk noise process, is about ± 100 percent, whereas that for the simulated flicker noise process is about ± 400 percent. On the other hand, using the same data length, estimation uncertainty of an Allan variance at 6-h sampling time for the simulated random walk noise process is about ± 40 percent, and that for the simulated flicker noise process is about ± 100 percent.

In our experiments shown in Fig. 2, we have taken the data for a period of four days. Thus the statistical measure of stabilities for the solid state noise source and the total-power radiometer have an uncertainty of ± 100 percent for a 1-day sampling time interval, whereas the statistical measure of the stability for the gas discharge noise source has an uncertainty of ± 300 percent for a 1-day sampling time interval. Of course, as the sampling time interval, τ, decreases, an uncertainty for estimating a statistical measure of a stability improves. The confidence levels (3σ) for our measurements at various sampling time intervals are also indicated in Fig. 2.

It is found from these exercises, that since the gain fluctuation of the total-power radiometer has a random walk spectrum, the use of the cross correlation technique for estimation of an individual instability is approximately valid. However, since the confidence in estimating an individual stability goes down as the number of samples decreases, the data length should be considerably larger than any sampling time intervals of interest, in order for this technique to be very useful.

VI. Summary and Conclusions

A concise determination for the measurement of stability has been given using an Allan variance analysis. The measure was applied to the evaluation of the stability of the output radiation noise temperature of solid state noise sources. It is found that the square root of the variance for the output noise from a typical solid state noise source for a 1-day sampling time interval is typically 0.0014, or ± 0.006 dB; however, because of estimation uncertainty, it could be 0.0028, or ± 0.012 dB. In contrast, the square root of the variance for the output noise from a typical argon gas discharge noise source for a 1-day sampling time interval is about 0.0002, or ± 0.001 dB; however, because of estimation

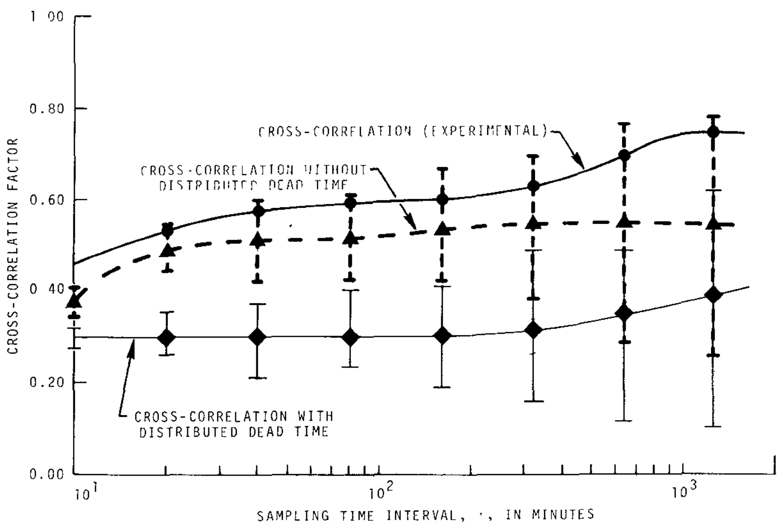

Fig. 8. Cross correlations of Allan variances using data with and without distributed dead time.

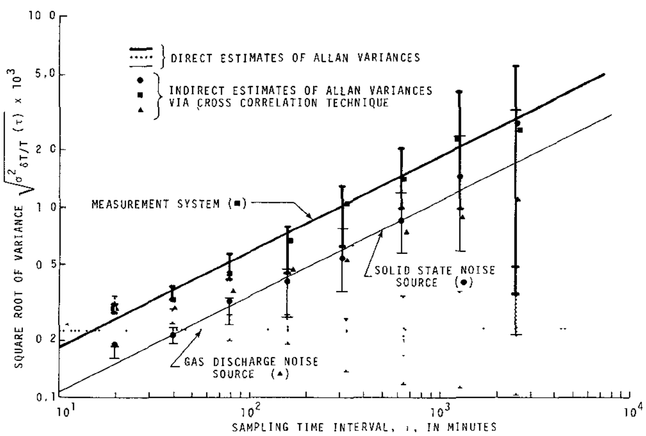

Fig. 9. Comparison between direct estimates of Allan variances and indirect estimates of Allan variances derived from cross correlation technique.

uncertainty, it could be 0.0008, or ± 0.004 dB. A cross correlation technique and a triangulation technique were employed to estimate the stability of individual noise sources. Although Allan variance estimation from data containing distributed dead time has little bias, the estimation of cross correlation from such data has significant bias.

A brief study indicates that the operating current of a solid state noise source is one of the important environmental factors which influence the stable operation of the solid state noise source.

It is found, therefore, that the stability of a typical commercial solid state noise source is not as good as that of a typical commercial argon gas noise source. However, by implementing such modifications as 1) heat sinking of a silicon avalanche noise diode, 2) proper dc RF decoupling, and 3) impedance matching, the stability of the NBS solid state noise source has been improved significantly over that of typical commercial solid state noise sources. Implementation and the resulting improvement of these modifications were discussed elsewhere by the author [8], [9].

It is emphasized that this measure of stability, using Allan variance analysis, allows one to specify the performance of solid state noise sources and facilitates more meaningful, relative comparisons among these devices than was previously possible.

Acknowledgment

The author is indebted to D. W. Allan and D. Halford for many stimulating discussions and to T. H. Bremer for his experimental assistance. Appreciation is also extended to C. K. S. Miller and D. F. Wait for their support and encouragement.

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